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Pure spinor superfields and Born–Infeld theory

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Abstract: We present a method for introducing and analysing higher-derivative deformations of maximally supersymmetric field theories. Such terms are built in the pure spinor superfield framework, using a set of operators representing physical fields. The action for abelian Born–Infeld theory becomes polynomial in this language, and contains only a four-point interaction in addition to the free action. Simplifications also occur in the non-abelian case.

1. INTRODUCTION

Pure spinors have a long history in connection to (maximal) supersymmetry and superstring theory [1-9]. The main achievement of the formalism is that it solves the problem of manifesting supersymmetry, without breaking other symmetries, in maximally supersymmetric models.

Variations of the pure spinor formalism has been successfully used for calculation of perturbative amplitudes in maximally supersymmetric gauge theory and superstring theory [10-15]. A partly overlapping activity has been the search for “supersymmetric invariants”, *i.e.*, possible counterterms or terms in a low-energy effective action [6,7,16-19].

It has become clear during the past few years that the most efficient use of the pure spinor formalism (in field theory) is when the full Batalin–Vilkovisky (BV) field–antifield formalism [20,21] is used. The language is that of classical BV theory. The canonical example is $D = 10$ super–Yang–Mills (SYM) theory [22], which will be reviewed below, but also other maximally supersymmetric field theories, including supergravity, have been formulated in this framework [23,24]. The resulting actions generically are “simpler” than the component ones, and contain terms with lower maximal degree of homogeneity. In certain cases, an action which is non-polynomial in physical component fields becomes polynomial [24]. This of course inspires to ask similar question concerning supersymmetric invariants in general: Can they show similar simplifications?

An important example of a supersymmetric invariant, possessing some remarkable properties, and relevant in string theory as the effective theory of supersymmetric D-branes [25-29], is the Born–Infeld theory [30]. Much work has been done on Born–Infeld theory, its supersymmetric versions, and its rôle in string theory, see *e.g.* refs. [31-42]. While the abelian Born–Infeld Lagrangian (and its supersymmetrisation) is well known, the non-abelian counterpart, describing coincident D-branes [43], has been elusive. The full term at order α'^2 (the “ F^4 term”) was first constructed in ref. [7], and extended to the next order (α'^4) in ref. [44]. No closed expression has been proposed, consistent with the known terms to this order.

One main motivation for the present investigation is to examine whether Born–Infeld dynamics exhibits any simplification in the pure spinor superfield formalism. We will see that this is indeed the case. The abelian Born–Infeld action becomes polynomial, with only an α'^2 term in addition to the kinetic term, and consequently the α'^2 term contains all information on the symmetrised trace part in the non-abelian situation.

Born–Infeld theory has been considered earlier in connection to pure spinors in ref. [45], where pure spinor superstrings ending on a D-brane were constructed. Unlike our treatment, where non-minimal variables are needed, the obtained superspace equations of motion are contained in a minimal pure spinor setting, and are non-polynomial.

Another motivation is the systematic search for invariants in general. Collinucci, de Roo and Eenink showed in ref. [46], using a component formalism, that there are two independent quartic deformations of $D = 10$ Maxwell theory. In addition to the one at order α'^2 given below, there is also one at order α'^4 . They also constructed a linear deformation of $D = 10$ super-Yang–Mills theory at order α'^3 whose lowest order terms are quartic [47]. There is no non-trivial deformation at α'^3 in Maxwell theory [47,48]. Movshev and Schwarz have given a classification of infinitesimal deformations of $D = 10$ super-Yang–Mills theory and its dimensional reductions [49,50]. We would like to connect to their results, but will leave this for the future.

Let us sketch our main line of thought. Earlier approaches to deformations (higher-derivative terms) in the pure spinor formalism have used only part of the information available, namely the equations for the physical superfields at ghost number 0. This is logical in a certain sense — although the cohomology contains also ghosts and antifields, the gauge transformations should not be deformed. But if we want to use the full BV formalism, which means that the master equation is what defines a consistent deformation, we need to retain the full pure spinor superfields. Still, deformations are built from physical fields, and we need a way to “obtain them” from the field. More specifically, we need to find operators of ghost number -1 that act on the pure spinor superfield Ψ and give new pure spinor superfields of ghost number 0, transforming as physical fields. We also need to understand the cohomology of such fields. In refs. [6,7], the α'^2 terms was encoded in the superspace relation

$$F_{\alpha\beta}^A \sim \alpha'^2 t^A{}_{BCD} (\gamma^i \chi^B)_\alpha (\gamma^j \chi^C)_\beta F_{ij}^D, \quad (1.1)$$

where χ is the physical fermion, and t_{ABCD} a totally symmetric tensor. The physical fields on the right hand side need to be reexpressed using operators of negative ghost number.

Interesting results have been obtained using harmonic superspace (see *e.g.* refs. [51,52] and references therein). We will not try to make any comparison between such approaches and the one advocated in the present paper.

The paper is organised as follows. In Section 2, the pure spinor superfield BV formalism for $D = 10$ SYM is briefly reviewed. Section 3 deals with non-scalar superfields and their gauge symmetries. The operators corresponding to physical fields are derived in Section 4, and used in Section 5 to construct interaction terms respecting the BV master equation. Our results are summarised in Section 6, where we also propose further lines of exploration. Some conventions are given in Appendix A. A few useful spinor identities are listed in Appendix B, and Appendix C gives the list of modules appearing in polynomials in the non-minimal pure spinor variables.

2. BATALIN–VILKOVISKY ACTIONS FOR PURE SPINOR FIELDS

It is well known that the superspace formulation of $D = 10$ super-Yang–Mills theory is contained in a scalar fermionic pure spinor superfield $\Psi(x, \theta; \lambda)$ of ghost number 1, subject to the equation of motion

$$Q\Psi + \Psi^2 = 0 . \quad (2.1)$$

Here, λ^α is a pure spinor, *i.e.*, $(\lambda\gamma^a\lambda) = 0$. The BRST operator Q is defined as $Q = (\lambda D)$, D_α being the fermionic covariant derivative. (We also often suppress a Lie algebra index on the field. For example, $(\Psi^2)^A = \frac{1}{2}f^A{}_{BC}\Psi^B\Psi^C$, where f are structure constants.)

This is the canonical example of a pure spinor superfield theory. The other example with a scalar field is $D = 11$ supergravity [24]. Much of what is said here applies also to that model.

The standard on-shell superspace description is obtained by restricting to a superfield of ghost number 0, $\Psi(x, \theta; \lambda) = \lambda^\alpha A_\alpha(x, \theta)$. The equation of motion then is the 5-form part of the superspace flatness condition [53]

$$F_{\alpha\beta} \equiv 2D_{(\alpha}A_{\beta)} + \{A_\alpha, A_\beta\} + 2\gamma^i_{\alpha\beta}A_i = 0 . \quad (2.2)$$

The vector part of this equation is the usual conventional constraint.

In addition to describing the physical fields, the solution of equation (2.1) gives the ghost field at ghost number 1, and the antifields of the physical and ghost fields at ghost number -1 and -2 , respectively. The natural language becomes the field–antifield formalism of Batalin and Vilkovisky [20,21].

With an appropriately defined integration, the equations of motion come from a Chern–Simons-like action

$$S_{\text{CS}} = \int [dZ] \text{tr} \left(\frac{1}{2} \Psi Q \Psi + \frac{1}{3} \Psi^3 \right) , \quad (2.3)$$

and one will then have an supersymmetric off-shell formulation of the theory. Defining the integration (and regularising integrals) calls for the introduction of non-minimal pure spinor variables [54]. In addition to the pure spinor λ^α , a pure spinor $\bar{\lambda}_\alpha$ of the opposite chirality is introduced, together with a fermionic spinor r_α satisfying $(\bar{\lambda}\gamma^a r) = 0$. The BRST operator is extended to

$$Q = (\lambda D) + \left(r \frac{\partial}{\partial \lambda} \right) , \quad (2.4)$$

which leaves its cohomology unchanged. We refer to refs. [54,11] for details.

How is the consistency of the action (2.3) checked? This is especially relevant when we will try to add higher-derivative terms. One may check that the action is invariant under

gauge transformations $\delta\Psi = Q\Lambda + [\Psi, \Lambda]$. But a more efficient way is to take the Batalin–Vilkovisky formalism *ad notam*. In this framework, the action itself is the generator of “gauge transformations” in a generalised sense, via the antibracket. Invariance of the action itself is encoded in the master equation

$$(S, S) = 0 , \quad (2.5)$$

which is also the appropriate generalisation of “ $Q^2 = 0$ ” to an interacting theory. The antibracket in pure spinor field theory takes the simplest possible form [24]:

$$(A, B) = \int A \frac{\overleftarrow{\delta}}{\delta\Psi^A(Z)} [dZ] \frac{\overrightarrow{\delta}}{\delta\Psi^A(Z)} B . \quad (2.6)$$

The field Ψ is self-conjugate with respect to the antibracket.

This makes the master equation reasonably easy to check, also when we later add terms corresponding to higher-derivative corrections. These terms (seen as infinitesimal deformations) also have a clear cohomological interpretation: they are additional terms S' in the action, with the property of being closed, $(S, S') = 0$ but counted modulo exact terms (corresponding to field redefinitions), $S' \approx S' + (S, R)$.

3. NON-SCALAR FIELDS AND SHIFT SYMMETRIES

The basic field in $D = 10$ (or $D = 4$, $N = 4$) super–Yang–Mills theory is scalar (because the ghost is a scalar). BRST cohomology arises thanks to the pure spinor constraint. For example, consider the zero mode of the gauge connection. It sits in the field as $\Psi \sim (\lambda\gamma^i\theta)A_i$. Acting with Q gives the pure spinor constraint. It is also clear that no other (zero mode) cohomology than the ghost cohomology $\Psi \sim 1$ exists at λ^0 . But what about fields of ghost number zero, describing supermultiplets? In some cases (though not in $D = 10$) one needs to describe hypermultiplets, which have no gauge invariance. It they are to be described by a pure spinor superfield, which we may call Φ^I , the field will need to have ghost number 0 and come in the same module (of Lorentz and R-symmetry) as the scalar component fields. In addition, one needs the fermionic fields to be represented by cohomology at $\lambda^0\theta^1$. It is clear that this cannot be achieved by simply demanding $Q\Phi^I = 0$. This cohomology will simply be the tensor product of the cohomology of a scalar field with the module of Ψ^I , and there is no room for the fermions. In $D = 10$, although no non-scalar field is needed to describe the SYM multiplet, we have argued in the introduction that it will be necessary to introduce derived fields transforming in the modules of physical fields. This construction will be performed in the following Section. So, the situation is similar. Some other ingredient than the pure spinor constraint is needed.

Suppose that, in addition to the pure spinor constraint, which should be seen as a gauge symmetry defining the equivalence class $\Psi \approx \Psi + (\lambda \gamma^i \lambda) \Xi_i$, one has a further gauge symmetry involving the index structure of the field. Suppose, in the example with a hypermultiplet above (although the argument is completely general), that we have scalars ϕ^I and fermions $\chi^{\tilde{\alpha}}$. χ should come at level θ in a superfield starting with ϕ ,

$$\Phi^I(x, \theta) = \phi^I(x) + R_{\alpha\tilde{\alpha}}^I \theta^\alpha \chi^{\tilde{\alpha}} + \dots \quad (3.1)$$

(the invariant tensor R is typically a γ -matrix). Now, we demand that (at least the zero mode of) the fermion arises as pure spinor BRST cohomology. Acting with Q on the superfield (3.1) gives

$$Q\Phi^I \sim R_{\alpha\tilde{\alpha}}^I \lambda^\alpha \chi^{\tilde{\alpha}} + \dots \quad (3.2)$$

The fermions will represent cohomology only if this expression is “zero”, *i.e.*, if fields are taken in the equivalence class

$$\Phi^I \approx \Phi^I + R_{\alpha\tilde{\alpha}}^I \lambda^\alpha \xi^{\tilde{\alpha}} \quad (3.3)$$

We call this a “shift symmetry”. It should be thought of on an equal footing as the pure spinor constraint.

Such constructions have been relevant for the supersymmetric descriptions of BLG and ABJM models in $D = 3$ [23]. It was also essential in $D = 11$ supergravity [24], where the vector field Φ^a corresponding to the superspace geometry was constructed as an operator R^a acting on the fundamental scalar field Ψ corresponding to the tensor field.

As already demonstrated by the supergravity application, the principle is not only relevant for describing matter multiplets. In the following Section, we will show how to reinterpret the superspace Bianchi identities in terms of pure spinor superfields of ghost number 0, which will be obtained from Ψ by acting with operators of ghost number -1 with certain properties interpreted as shift symmetry.

This will provide the concrete answer to the question posed in the Introduction how the ghost number 0 superfields can be extended to pure spinor fields and used to construct deformation terms in the action.

4. PHYSICAL OPERATORS

4.1. FROM SUPERSPACE TO OPERATORS

Let us go back to the superspace equations of motion (2.2), implied by eq. (2.1), which follow from the action

$$\begin{aligned} S_{\text{CS}} = S_2 + S_3 &= \int [dZ] \text{tr} \left(\frac{1}{2} \Psi Q \Psi + \frac{1}{3} \Psi^3 \right) \\ &= \int [dZ] \left(\frac{1}{2} \Psi^A Q \Psi^A + \frac{1}{6} f_{ABC} \Psi^A \Psi^B \Psi^C \right) . \end{aligned} \quad (4.1)$$

The standard superspace calculation (“solving the Bianchi identities”) reveals a sequence of ghost number 0 superfields, constrained to be related by successive fermionic covariant derivatives:

$$\begin{aligned} D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\} + 2\gamma_{\alpha\beta}^i A_i &= 0 , \\ F_{a\alpha} = \partial_a A_\alpha - D_\alpha A_a + [A_a, A_\alpha] &= (\gamma_a \chi)_\alpha , \\ \mathcal{D}_\alpha \chi^\beta \equiv D_\alpha \chi^\beta + \{A_\alpha, \chi^\beta\} &= \frac{1}{2} (\gamma^{ij})_\alpha{}^\beta F_{ij} , \\ \mathcal{D}_\alpha F_{ab} \equiv D_\alpha F_{ab} + [A_\alpha, F_{ab}] &= 2(\gamma_{[a} \eta_{b]})_\alpha , \\ \dots \end{aligned} \quad (4.2)$$

Here, χ^α is the physical fermion superfield, and A_a the gauge connection superfield. All fields will of course be on shell. The field denoted η_a^α is the covariant derivative of χ^a : $\eta_a^\alpha = \mathcal{D}_a \chi^\alpha$, which is γ -traceless on shell.

As discussed in the Introduction, we need to interpret these fields as ghost number 0 parts of pure spinor superfields. In order to find such fields, we need to reinterpret the equations (4.2) as equations where one power of λ has been stripped off, not two. Contracting with *one* λ , and again denoting $(\lambda A) = \Psi$ gives:

$$\begin{aligned} D_\alpha \Psi + Q A_\alpha + \{\Psi, A_\alpha\} + 2(\gamma^i \lambda)_\alpha A_i &= 0 \\ \partial_a \Psi - Q A_a + [A_a, \Psi] &= (\lambda \gamma_a \chi) \\ Q \chi^\alpha + \{\Psi, \chi^\alpha\} &= -\frac{1}{2} (\gamma^{ij} \lambda)^\alpha F_{ij} , \\ Q F_{ab} + [\Psi, F_{ab}] &= 2(\lambda \gamma_{[a} \eta_{b]}) , \\ \dots \end{aligned} \quad (4.3)$$

Note that (as soon as one gets to gauge covariant fields) these identities are non-linear versions of cohomology modulo shift symmetry described in Section 3.

We would like to interpret these as relations with the entire field Ψ , not only the ghost number 0 part. Therefore, we need to replace the ghost number zero superfields with pure spinor superfields, constructed as some operators with ghost number -1 acting on Ψ , such that the equations (4.3) hold when $Q\Psi + \Psi^2 = 0$. We thus make an Ansatz

$$\begin{aligned} A_\alpha &= \hat{A}_\alpha \Psi , \\ A_a &= \hat{A}_a \Psi , \\ \chi^\alpha &= \hat{\chi}^\alpha \Psi , \\ F_{ab} &= \hat{F}_{ab} \Psi , \\ &\dots \end{aligned} \tag{4.4}$$

Note that, since Ψ is fermionic, the operators have opposite statistics to the corresponding fields. An almost immediate inspection gives at hand that the equations (4.3) follow from $Q\Psi + \Psi^2 = 0$ given that the operators satisfy

$$\begin{aligned} [Q, \hat{A}_\alpha] &= -D_\alpha - 2(\gamma^i \lambda)_\alpha \hat{A}_i , \\ \{Q, \hat{A}_a\} &= \partial_a - (\lambda \gamma_a \hat{\chi}) , \\ [Q, \hat{\chi}^\alpha] &= -\frac{1}{2}(\gamma^{ij} \lambda)^\alpha \hat{F}_{ij} , \\ \{Q, \hat{F}_{ab}\} &= 2(\lambda \gamma_{[a} \hat{\eta}_{b]}) , \\ &\dots \end{aligned} \tag{4.5}$$

It was therefore consistent to make a linear Ansatz. These expressions can be solved sequentially. Note that the shift term (in the sense defined in Section 3) occurring in the equation stating the closedness of one operator defines the next one, again modulo a shift transformation. The operator equation can be solved explicitly, but, as usual when it comes to constructing operators with negative ghost number, one needs to use non-minimal pure spinor variables. The result is

$$\begin{aligned} \hat{A}_\alpha &= -(\lambda \bar{\lambda})^{-1} \left[\frac{1}{8}(\gamma^{ij} \bar{\lambda})_\alpha N_{ij} + \frac{1}{4} \bar{\lambda}_\alpha N \right] , \\ \hat{A}_a &= -\frac{1}{4}(\lambda \bar{\lambda})^{-1} (\bar{\lambda} \gamma_a D) + \frac{1}{32}(\lambda \bar{\lambda})^{-2} (\bar{\lambda} \gamma_a{}^{ij} r) N_{ij} , \\ \hat{\chi}^\alpha &= \frac{1}{2}(\lambda \bar{\lambda})^{-1} (\gamma^i \bar{\lambda})^\alpha \partial_i - \frac{1}{192}(\lambda \bar{\lambda})^{-2} (\bar{\lambda} \gamma^{ijk} r) (\gamma_{ijk} D)^\alpha \\ &\quad - \frac{1}{64}(\lambda \bar{\lambda})^{-3} (\gamma_i \bar{\lambda})^\alpha (r \gamma^{ijk} r) N_{jk} , \\ \hat{F}_{ab} &= \frac{1}{8}(\lambda \bar{\lambda})^{-2} (\bar{\lambda} \gamma_{ab}{}^i r) \partial_i + \frac{1}{32}(\lambda \bar{\lambda})^{-3} (r \gamma_{ab}{}^i r) (\bar{\lambda} \gamma_i D) \\ &\quad - \frac{1}{256}(\lambda \bar{\lambda})^{-4} (\bar{\lambda} \gamma_{abi} r) (r \gamma^{ijk} r) N_{jk} , \\ &\dots \end{aligned} \tag{4.6}$$

(Here, $N^{ab} = (\lambda\gamma^{ab}w)$ and $N = (\lambda w)$, with $w_\alpha = \frac{\partial}{\partial\lambda^\alpha}$.) These expressions are not strictly unique. Each operator should be considered as a representative of an equivalence class, modulo exact terms and shift terms. Some spinor identities that are useful in this calculation are given in Appendix B. Appendix C contains information on polynomials in $\bar{\lambda}$ and r which has also been of help.

Some later calculations can be simplified by noting that the physical fermion operator can be written

$$\hat{\chi}^\alpha = \frac{1}{2}(\lambda\bar{\lambda})^{-1}(\gamma^i\bar{\lambda})^\alpha\Delta_i, \quad (4.7)$$

where the modified derivative Δ_a is defined as

$$\Delta_a = \partial_a + \frac{1}{4}(\lambda\bar{\lambda})^{-1}(r\gamma_a D) - \frac{1}{32}(\lambda\bar{\lambda})^{-2}(r\gamma_{aij}r)N^{ij}. \quad (4.8)$$

In addition, the field strength operator is expressible in terms of the fermion operator, and therefore in terms of Δ , as

$$\hat{F}_{ab} = -\frac{1}{4}(\lambda\bar{\lambda})^{-1}(r\gamma_{ab}\hat{\chi}) = \frac{1}{8}(\lambda\bar{\lambda})^{-2}(\bar{\lambda}\gamma_{ab}{}^i r)\Delta_i. \quad (4.9)$$

The Q -closedness modulo shifts of the physical operators is guaranteed by the BRST transformation of Δ_a :

$$[Q, \Delta_a] = \frac{1}{2}(\lambda\bar{\lambda})^{-1}(r\gamma_a\gamma^i\lambda)\Delta_i. \quad (4.10)$$

It is straightforward to reverse the whole argument and verify that when these operators act on a field $\Psi = (\lambda A)$ satisfying $Q\Psi + \Psi^2 = 0$, they will give the physical superfields (modulo shifts). It is important to realise that the on-shell meaning of *e.g.* $\hat{F}_{ab}\Psi$ will depend on which the equations of motion are. Consider for example the difference between the abelian and non-abelian case in eq. (4.4). When we, in the following Section, deform the equations of motion, the on-shell meaning of these fields will obviously change.

4.2. OPERATOR IDENTITIES

It is possible to take the correspondence one step further even off-shell, using the defining properties (4.5) of the operators. When we use an operator like \hat{F}_{ab} in the construction of some term in the action, it is important to verify that they are cohomologically equivalent to a derivative together with some other operator. Otherwise, the counting of deformations may go wrong. Let us start with the next-to-lowest operator, \hat{A}_a . It should in some sense be equivalent with a fermionic derivative on \hat{A}_α , since this is how the physical fields behave. In order to show this, we may use the shift symmetries. As mentioned, a shift transformation

of one operator in the sequence leads to a BRST transformation of the following. Consider a finite shift of \hat{A}_α of the form

$$\delta \hat{A}_\alpha = c(\gamma^i \lambda)_\alpha (\hat{A} \gamma_i \hat{A}) , \quad (4.11)$$

where c is a dimensionless constant. In order to calculate the corresponding change in \hat{A}_a (modulo a shift), we insert the first relation in eq. (4.5) to obtain

$$[Q, \delta \hat{A}_\alpha] = c(\gamma^i \lambda)_\alpha \left[-2(D\gamma_i \hat{A}) - 2(\gamma^j \lambda)_\beta \hat{A}_j (\gamma_i \hat{A})^\beta - 2(\gamma_i \hat{A})^\beta (\gamma^j \lambda)_\beta \hat{A}_j \right] \quad (4.12)$$

The operator \hat{A}_α satisfies

$$\begin{aligned} (\lambda \hat{A}) &= N , \\ (\lambda \gamma_{ab} \hat{A}) &= N_{ab} . \end{aligned} \quad (4.13)$$

Before these relations can be used, we need to order the operators in eq. (4.12). To this end, we use the relations

$$\begin{aligned} [\hat{A}_\alpha, \lambda^\beta] &= (\lambda \bar{\lambda})^{-1} \left[\frac{1}{8} (\gamma^{ij} \bar{\lambda})_\alpha (\gamma^{ij} \lambda)^\beta - \frac{1}{4} \bar{\lambda}_\alpha \lambda^\beta \right] , \\ [\hat{A}_a, \hat{A}_\alpha] &= (\lambda \bar{\lambda})^{-1} \bar{\lambda}_\alpha \hat{A}_a , \end{aligned} \quad (4.14)$$

which are derived from the explicit expressions (4.6) (we have a feeling that the second of these relations should have some more immediate interpretation, but are not aware of one). Using these commutators to order the terms in eq. (4.12) to enable use of eq. (4.13), we obtain, after some calculation, a simple expression for the trivial change in \hat{A}_a :

$$(\gamma^i \lambda)_\alpha \delta \hat{A}_i = -2c(\gamma^i \lambda)_\alpha \left[(D\gamma_i \hat{A}) + (4N + 16)\hat{A}_i \right] \quad (4.15)$$

So, anything proportional to the expression inside the square brackets is a trivial (BRST transformation plus shift) change of \hat{A}_i . It is straightforward to perform the consistency check that the right hand side of eq. (4.15) is closed due to eq. (4.5). Eq. (4.15) agrees with what is stated in the linearisation of the first equation in (4.2). This relation has now been lifted in a precise way to the operator level. Most importantly, the operator relation is valid off-shell.

This procedure can be continued to the higher-dimensional operators. A shift in \hat{A}_a with

$$\delta \hat{A}_a = c(\lambda \gamma_a \gamma^i \hat{A}) \hat{A}_i \quad (4.16)$$

leads to

$$\delta\hat{\chi}^\alpha = c \left[\partial_i (\gamma^i \hat{A})^\alpha - (\gamma^i D)^\alpha \hat{A}_i - (2N + 10) \hat{\chi}^\alpha \right] , \quad (4.17)$$

where we in the process needed to derive the relation $(\lambda \gamma_a \gamma^i \hat{A})(\gamma_i \lambda)_\alpha = (\gamma_a \lambda)_\alpha (2N + 10)$. Again, the trivial expression in the square brackets matches the linearised equation for the field χ^α .

Similar identities, relating the composition of one operator and a spinorial derivative to the following operator, can be derived analogously.

Shifts involving \hat{A}_a instead of \hat{A}_α directly give compositions with bosonic derivatives. Let us take one example, which also clarifies a technical issue with the shift symmetry we have yet refrained from mentioning. Consider a shift in $\hat{\chi}^\alpha$ of the form

$$\delta\hat{\chi}^\alpha = c(\gamma^{ij}\lambda)^\alpha \hat{A}_i \hat{A}_j . \quad (4.18)$$

In order to derive the ensuing trivial change in \hat{F}_{ab} , we need two interesting relations, namely,

$$\begin{aligned} [\hat{A}_a, (\lambda \gamma_b \hat{\chi})] &= -\hat{F}_{ab} , \\ \hat{A}^i (\lambda \gamma_i \hat{\chi}) &= b - \partial^i \hat{A}_i , \end{aligned} \quad (4.19)$$

where b is the “ b -ghost”, the gauge fixing operator [54] associated with Siegel gauge,

$$\begin{aligned} b &= -\frac{1}{2}(\lambda\bar{\lambda})^{-1}(\bar{\lambda}\gamma^i D)\partial_i + \frac{1}{16}(\lambda\bar{\lambda})^{-2}(\bar{\lambda}\gamma^{ijk}r)(N_{ij}\partial_k + \frac{1}{24}(D\gamma_{ijk}D)) \\ &\quad - \frac{1}{64}(\lambda\bar{\lambda})^{-3}(r\gamma^{ijk}r)(\bar{\lambda}\gamma_i D)N_{jk} - \frac{1}{1024}(\lambda\bar{\lambda})^{-4}(\bar{\lambda}\gamma^{ijm}r)(r\gamma^{kl}{}_m r)N_{ij}N_{jk} . \end{aligned} \quad (4.20)$$

This gives

$$[Q, \delta\hat{\chi}^\alpha] = c(\gamma^{ij}\lambda)^\alpha (2\partial_i \hat{A}_j - \hat{F}_{ij}) - 2c\lambda^\alpha (b - \partial^i \hat{A}_i) . \quad (4.21)$$

The trivial change in \hat{F}_{ab} is what one expects, but there is also a shift generated corresponding to a scalar. We see that it corresponds to a gauge degree of freedom. In fact, the gauge invariant entity $(\lambda \gamma_a \hat{\chi})$, appearing on the right hand side in eq. (4.5), allows not only for a shift $\delta\hat{\chi}^\alpha = (\gamma^{ij}\lambda)^\alpha \hat{\xi}_{ij}$, but also $\delta\hat{\chi}^\alpha = \lambda^\alpha \hat{\xi}$ (in fact, there is no shift invariant expression invariant under the first but not under the second transformation). While the first of these shifts corresponds to the presence of the field strength at level θ in the superfield χ^α , the second has nothing to do with physical degrees of freedom, but corresponds to longitudinal modes. The second term in eq. (4.21), and therefore the gauge type of shift in $\hat{\chi}^\alpha$, is necessary; the first term on the right hand side is not closed. In the process, we have shown that Siegel gauge implies Lorenz gauge.

We have not checked higher operators explicitly. Our impression is that there are no single-derivative operators that are cohomologically equivalent to $\partial\hat{\chi}$ or $\partial\hat{F}$. This would be as well, since the operators we have formulated (together with bosonic derivatives) suffice to write operators corresponding to all gauge-covariant physical fields in the theory.

One further comment on the form of the operators. Since any cohomology can be represented as a function of the minimal variables only (independent of $\bar{\lambda}$ and θ), one may be tempted to think that the r -independent terms of the operators are the relevant ones, and that *e.g.* \hat{F} is trivial. This is not the case. The operators map cohomology to cohomology, but they do not respect a gauge choice where there is no dependence on the non-minimal sector. Indeed, if one examines the action of \hat{A}_a on the zero mode $\Psi = -(\lambda\gamma^i\theta)A_i$, one will find that the two terms contribute equally to the cohomology. In $\hat{\chi}^\alpha$, only r -dependent terms contribute to the zero mode, etc. This is important to understand, so that one does not draw a conclusion implying that deformation terms containing \hat{F} (and, consequently, having no r -independent terms) would be trivial. The non-triviality of \hat{F}_{ab} is of course demonstrated by the above calculations.

The identities

$$\begin{aligned} [\hat{\chi}^\alpha, \hat{\chi}^\beta] &= 0 , \\ [\hat{\chi}^\alpha, \hat{F}_{ab}] &= 0 , \\ \{\hat{F}_{ab}, \hat{F}_{cd}\} &= 0 \end{aligned} \tag{4.22}$$

are straightforward to prove. Also the higher operators (anti-)commute. We also have

$$\begin{aligned} [\hat{\chi}^\alpha, (\lambda\gamma_a\hat{\chi})] &= 0 , \\ [(\lambda\gamma_a\hat{\chi}), (\lambda\gamma_b\hat{\chi})] &= 0 , \\ [(\lambda\gamma_a\hat{\chi}), \hat{F}_{bc}] &= 0 . \end{aligned} \tag{4.23}$$

In addition, $\hat{\chi}^\alpha$ is pure, $(\hat{\chi}\gamma^a\hat{\chi}) = 0$. We also have an identity

$$(\lambda\gamma^i\hat{\chi})\hat{F}_{ai} = 0 . \tag{4.24}$$

These identities are very helpful, and simplify calculations needed to check the master equation.

The construction in this Section has been performed for $D = 10$ super-Yang–Mills theory. It is of course straightforward to adapt it to $D = 4$, $N = 4$ SYM, where also some scalar fields become available as physical operators. An analogous construction should exist for $D = 11$ supergravity [24]. There, the “vielbein field” Φ^a , possessing a certain shift symmetry, was already given as an operator of ghost number -2 acting on the basic field

Ψ . Pure spinor superfields of ghost number 0 could be derived by further action of ghost number -1 operators on Φ^a . These fields may then be used in higher-derivative terms along the same lines as in the following Section.

5. BORN–INFELD AND OTHER DEFORMATIONS

Any term in the action should be expressed as an integral over all variables, where the integrand has ghost number 3 and dimension 0. Terms with higher derivatives of course carry some factors of α' to match dimension, but ghost number has to be respected.

A higher-derivative deformation should not change how gauge symmetry transforms physical component fields. But in a BV framework, gauge symmetries and equations of motion are inextricably integrated — the gauge symmetry of the antifields is the equations of motion for the fields and vice versa. Our basic field Ψ is self-conjugate with respect to the antibracket, so fields and antifields can not be separated. In order to ensure that a deformation starts with the equations of motion for the physical fields, *i.e.*, at order λ^2 , one may look for a representative of the deformation containing exactly two explicit powers of λ . Any term should respect shift symmetry, and it will be ensured by the explicit powers of λ . On the other hand, an expression which is a product of shift-invariant expressions runs the risk of being trivial in view of eq. (4.5). As we will see, this gives severe restrictions — non-trivial terms can arise when shift symmetry is achieved by the “sharing” of λ ’s between more than one field. Another principle, which we believe is true, although we do not have a strict proof, is that it is sufficient to consider expressions where not more than one operator acts on each field.

We will examine the deformation starting at α'^2 , *i.e.*, “ F^4 terms” and associated higher derivative terms, in the abelian and non-abelian settings.

5.1. THE ABELIAN CASE

Let us first consider abelian (Maxwell) theory. The α'^2 term is known, and given, at the level of equations of motion, by eq. (1.1). We now follow our program of replacing superfields by pure spinor superfields, obtained by letting the physical operators of Section 3 act on Ψ .

The lowest order deformation of the Maxwell action then is

$$S = S_2 + S_4 = \int [dZ] \left(\frac{1}{2} \Psi Q \Psi + \frac{k}{4} \Psi (\lambda \gamma^i \hat{\chi}) \Psi (\lambda \gamma^j \hat{\chi}) \Psi \hat{F}_{ij} \Psi \right) \quad (5.1)$$

where k is a constant proportional to α'^2 . It leads to the equations of motion

$$0 = Q\Psi + k(\lambda\gamma^i\hat{\chi})\Psi(\lambda\gamma^j\hat{\chi})\Psi\hat{F}_{ij}\Psi . \quad (5.2)$$

Alternatively, the 4-point coupling can be written

$$S_4 = \frac{k}{4} \int [dZ] g_{\alpha\beta\gamma} \Psi \hat{\chi}^\alpha \Psi \hat{\chi}^\beta \Psi \hat{\chi}^\gamma \Psi , \quad (5.3)$$

where $g_{\alpha\beta\gamma} = \frac{1}{4}(\gamma^i\lambda)_{[\alpha}(\gamma^j\lambda)_{\beta}(\gamma_{ij}r)_{\gamma]}$. Here, the commutativity of eqs. (4.22) and (4.23) has been used. An even simpler version, using eq. (4.7), is

$$S_4 = \frac{k}{32} \int [dZ] (\lambda\bar{\lambda})^{-2} (\bar{\lambda}\gamma^{ijk}r) \Psi \Delta_i \Psi \Delta_j \Psi \Delta_k \Psi , \quad (5.4)$$

We see that this gives the linear deformation (1.1) described in refs. [6,7].

We would like to comment on the closedness and non-triviality of the term S_4 . It is easily seen that there are no possible trivial terms (S_2, R) resulting in $\lambda^2\Psi\chi^2F$ terms in the abelian case. We note that the action (5.1) has been written in a form where the $\hat{\chi}$'s enter in their shift-invariant form $(\lambda\gamma^a\hat{\chi})$, which is closed, but not exact. The \hat{F} factor, on the other hand, is not paired with a λ . The shift-invariant combination with \hat{F} , $(\gamma^{ij}\lambda)^\alpha\hat{F}_{ij}$, is BRST-exact, due to eq. (4.5), so having one λ for each operator would lead to a trivial expression. Associating the λ 's with the $\hat{\chi}$'s to form shift-invariant expressions is however a matter of taste. One could equally well write S_4 as proportional to

$$\Psi(\lambda\gamma^i\hat{\chi})\Psi(\hat{\chi}\Psi\gamma_i[Q, \hat{\chi}]\Psi) , \quad (5.5)$$

by instead associating one of the λ 's with \hat{F} , but in the process leaving one of the χ 's “naked”. The key to S_4 being closed but not exact is the fact that the λ 's are “shared” between fields to guarantee shift symmetry. The invariance of S_4 on the form (5.4) is less obvious, and of course relies on the transformation of the “prefactor”.

It turns out that not only does the action (5.1) define a consistent infinitesimal deformation, *i.e.*, $(S_2, S_4) = 0$, but it also fulfills the full master equation. The identity $(S_4, S_4) = 0$ can be seen by writing it as

$$\begin{aligned} (S_4, S_4) &= k^2 \int [dZ] (\lambda\gamma^a\hat{\chi})\Psi(\lambda\gamma^b\hat{\chi})\Psi\hat{F}_{ab}\Psi(\lambda\gamma^c\hat{\chi})\Psi(\lambda\gamma^d\hat{\chi})\Psi\hat{F}_{cd}\Psi \\ &= -\frac{k^2}{64} \int [dZ] (\lambda\bar{\lambda})^{-4} (\bar{\lambda}\gamma^{abc}r)(\bar{\lambda}\gamma^{ijk}r) \Delta_a \Psi \Delta_b \Psi \Delta_c \Psi \Delta_i \Psi \Delta_j \Psi \Delta_k \Psi . \end{aligned} \quad (5.6)$$

The only non-vanishing modules in $\bar{\lambda}^2 r^2$ are $(01011) \oplus (00110)$ (see Appendix C). The first of these is the one with six vector indices, a traceless tensor with symmetry \boxplus . The completely antisymmetric tensor, demanded in eq. (5.6), does not occur, so that expression vanishes identically.

The conclusion is that the action (5.1), with only a 4-point coupling in addition to the kinetic term, actually encodes the full abelian Born–Infeld theory. This may be seen as somewhat surprising, but is very much in line with the simplifications generically occurring in interacting pure spinor field theories [23,24]. It will be interesting, and maybe also instructive as a toy model for supergravity, to try to find the most efficient way of extracting the Born–Infeld dynamics for the physical component fields from the polynomial action (5.1). In both cases, the non-linearities generate square roots of determinants.

5.2. THE NON-ABELIAN CASE

How does the master equation work in the non-abelian case? We now depart from the action (2.3). A 4-point coupling can be written as

$$S_4 = \frac{1}{4} t_{ABCD} \int [dZ] \Psi^A (\lambda \gamma^i \hat{\chi}) \Psi^B (\lambda \gamma^j \hat{\chi}) \Psi^C \hat{F}_{ij} \Psi^D , \quad (5.7)$$

where $t_{ABCD} = t_{(ABCD)}$ is a symmetric invariant tensor in adjoint indices. It is easily shown, by expressing the operators in terms of the Δ operator, that other symmetries in t_{ABCD} do not occur.

The experience from Maxwell theory indicates that $S = S_{\text{CS}} + S_4$ satisfies the master equation, apart from resulting terms which are not completely symmetric in the adjoint indices. Remaining terms, which contain antisymmetrisations in adjoint indices, must be compensated by (S_2, S_6) . (S_2, S_4) is calculated as in the abelian case. $(S_3, S_4) = 0$ encodes the invariance of t_{ABCD} , *i.e.*, the identity $f_{A(B}{}^F t_{CDE)F} = 0$. The calculation for (S_4, S_4) works similarly as is the abelian case, but unlike in the abelian case, we can not assume that the tensor $t_{ABC}{}^G t_{DEFG}$ is completely symmetric.

The result now is

$$\begin{aligned} (S_4, S_4) = & -\frac{1}{64} \int [dZ] t_{ABC}{}^L t_{IJKL} (\lambda \bar{\lambda})^{-4} (\bar{\lambda} \gamma^{abc} r) (\bar{\lambda} \gamma^{ijk} r) \\ & \times \Delta_a \Psi^A \Delta_b \Psi^B \Delta_c \Psi^C \Delta_i \Psi^I \Delta_j \Psi^J \Delta_k \Psi^K , \end{aligned} \quad (5.8)$$

and we notice that this projects on the symmetry $\boxplus \boxplus$ in the tensor $t_{ABC}{}^G t_{DEFG}$. The right hand side of eq. (5.8) is not a total derivative. Six-point (and presumably higher) terms are

needed. We have not calculated such additional terms, and have so far not been able to deduce their structure using the properties of the operators in Section 4. We note that the entire contribution with symmetrised traces is encoded in S_4 .

6. CONCLUSIONS AND OUTLOOK

We have presented a method for lifting linear deformations constructed earlier in the superspace/pure spinor formulation to a consistent BV framework. The construction has a simple intuitive meaning in terms of operators corresponding to physical fields. These operators turned out to have remarkably simple properties. Most notably, the by now quite common phenomenon that pure spinor superspace BV actions reduce the degree of interactions is at work, to the degree that the abelian Born–Infeld action to all orders is represented by a kinetic term and a 4-point interaction. The non-abelian deformation also simplifies, in that the 4-point term encodes the full contribution from symmetrised traces.

Many question arise, which we have not yet addressed:

How are the equations of motion for the component field extracted? Even though all (linear) cohomology has representatives independent of the non-minimal variables $\bar{\lambda}$ and r , this will not be true for the interacting theory. The minimal form of the Born–Infeld superspace equations of motion obtained in ref. [45] might serve as a guideline. We hope that a systematic investigation of how Born–Infeld component equations of motion are reproduced can shed some light also on the nature of the polynomial action for $D = 11$ supergravity [24].

It is conceivable that our framework is efficient for finding more general supersymmetric invariants. It would be interesting to investigate this issue, both for $D = 10$ SYM and for its reduction to $D = 4$, where more possibilities may arise, and compare with refs. [49,50].

A similar construction should be performed in $D = 11$ and $D = 4$, $N = 8$ supergravity. We would also like to understand how to implement U-duality in our formalism, since it seems to have an important rôle to play in the investigation of supergravity counterterms (for recent considerations of invariants in supergravity, see *e.g.* refs. [55,56]).

APPENDIX A: CONVENTIONS AND NOTATION

All calculations are performed in flat space or flat superspace. Lorentz indices are denoted a, b, \dots or i, j, \dots , while chiral spinor indices are α, β, \dots . Lie algebra (adjoint) indices are A, B, \dots .

Flat superspace covariant derivatives in inertial basis are denoted D_α (fermionic) and ∂_a (bosonic).

Contractions of spinor indices are denoted (...), *e.g.* $(\lambda\gamma^a\chi) \equiv \lambda^\alpha\gamma_{\alpha\beta}^a\chi^\beta$.

Commutators and anticommutators of operators and fields are denoted with the same symbols, $[\cdot, \cdot]$ and $\{\cdot, \cdot\}$ respectively. The distinction should be clear from the context. When this denotes a commutator of fields (two bosonic or one bosonic and one fermionic) or an anticommutator of two fermionic fields, the notation is shorthand for (in the first case) $[U, V]^A \equiv f^A_{BC}U^BV^C$, not only for matrix algebras. Similarly the square of a fermionic Lie algebra valued field means $(\Psi^2)^A \equiv \frac{1}{2}f^A_{BC}\Psi^B\Psi^C$.

Batalin–Vilkovisky antibrackets are denoted (\cdot, \cdot) .

APPENDIX B: SOME USEFUL IDENTITIES

The antisymmetric product of three spinors is a γ -traceless 2-form spinor of the opposite chirality, $\wedge^3(00001) = (01010)$. This is manifested in the identity

$$\theta^\alpha(\theta\gamma_{abc}\theta) = \frac{1}{2}(\gamma_{[a}\gamma^i\theta)^\alpha(\theta\gamma_{bc]i}\theta) , \quad (B.1)$$

where the normalisation is determined by taking the γ -trace.

Some useful relations involving the pure spinor λ and the invariant combinations $N^{ab} \equiv (\lambda\gamma^{ab}w)$ and $N \equiv (\lambda w)$ are:

$$\begin{aligned} (\gamma^j\lambda)_\alpha N_{ij} &= (\gamma_i\lambda)_\alpha N , \\ (\gamma^{ij}\lambda)^\alpha N_{ij} &= 10\lambda^\alpha N , \\ (\gamma^{ij}\gamma_{abc}\lambda)_\alpha N_{ij} &= -2(\gamma_{abc}\lambda)_\alpha N - 24(\gamma_{[a}\lambda)_\alpha N_{bc]} . \end{aligned} \quad (B.2)$$

Some of the algebraic calculations involving spinors have been facilitated by the use of the Lie algebra program LiE [57] and the Mathematica package GAMMA [58].

APPENDIX C: PARTITION FUNCTIONS FOR THE NON-MINIMAL VARIABLES

In many of the calculations, it is practical to know the Lorentz modules appearing in some monomial $\bar{\lambda}^n r^m$. The LiE code given at the end of this appendix defines a function that does precisely this. The result may be summarised in a table:

	$\bar{\lambda}^0$	$\bar{\lambda}^1$	$\bar{\lambda}^{n \geq 2}$
r^0	(00000)	(00010)	(000 n 0)
r^1	(00010)	(00020) \oplus (00100)	(000, $n+1$,0) \oplus (001, $n-1$,0)
r^2	(00100)	(00110) \oplus (01001)	(001 n 0) \oplus (010, $n-1$,1)
r^3	(01001)	(01011) \oplus (02000) \oplus (10002)	(010 n 1) \oplus (020, $n-1$,0) \oplus (100, $n-1$,2)
r^4	(02000) \oplus (10002)	(00003) \oplus (02010) \oplus (10012) \oplus (11001)	(000, $n-1$,3) \oplus (020 n 0) \oplus (100 n 2) \oplus (110, $n-1$,1)
r^5	(00003) \oplus (11001)	(00013) \oplus (01002) \oplus (11011) \oplus (20100)	(000 n 3) \oplus (010, $n-1$,2) \oplus (110 n 1) \oplus (201, $n-1$,0)
r^6	(01002) \oplus (20100)	(01012) \oplus (10101) \oplus (20110) \oplus (30010)	(010 n 2) \oplus (101, $n-1$,1) \oplus (201 n 0) \oplus (300 n 0)
r^7	(10101) \oplus (30010)	(00200) \oplus (10111) \oplus (20011) \oplus (30020) \oplus (40000)	(002, $n-1$,0) \oplus (101 n 1) \oplus (200 n 1) \oplus (300, $n+1$,0)
r^8	(00200) \oplus (20011) \oplus (40000)	(00210) \oplus (10110) \oplus (20021) \oplus (30001)	(002 n 0) \oplus (101 n 0) \oplus (200, $n+1$,1)
r^9	(10110) \oplus (30001)	(01020) \oplus (10120) \oplus (20100)	(010, $n+1$,0) \oplus (101, $n+1$,0)
r^{10}	(01020) \oplus (20100)	(00030) \oplus (01030) \oplus (11010)	(000, $n+2$,0) \oplus (010, $n+2$,0)
r^{11}	(00030) \oplus (11010)	(00040) \oplus (02000) \oplus (10020)	(000, $n+3$,0)
r^{12}	(02000) \oplus (10020)	(01010)	\emptyset
r^{13}	(01010)	(00100)	\emptyset
r^{14}	(00100)	(00001)	\emptyset
r^{15}	(00001)	(00000)	\emptyset
r^{16}	(00000)	\emptyset	\emptyset

We define partition functions $P_m(t)$ counting the number of states at a given power r^m and arbitrary power of $\bar{\lambda}$. P_0 is the usual pure spinor partition function. The complete set

of functions is

$$\begin{aligned}
P_0(t) &= (1-t)^{-11}(1+5t+5t^2+t^3) , \\
P_1(t) &= (1-t)^{-11}(16+70t+46t^2) , \\
P_2(t) &= (1-t)^{-11}(120+440t+110t^2-10t^3) , \\
P_3(t) &= (1-t)^{-11}(560+1600t-416t^2+446t^3-330t^4+165t^5 \\
&\quad -55t^6+11t^7-t^8) , \\
P_4(t) &= (1-t)^{-11}(1820+3500t-3460t^2+4620t^3-4620t^4+3300t^5 \\
&\quad -1650t^6+550t^7-110t^8+10t^9) , \\
P_5(t) &= (1-t)^{-11}(4368+3640t-9800t^2+20608t^3-29040t^4+28578t^5 \\
&\quad -19910t^6+9680t^7-3136t^8+610t^9-54t^{10}) , \\
P_6(t) &= (1-t)^{-11}(8008-66440t+269434t^2-656238t^3+1016400t^4-965184t^5 \\
&\quad +408870t^6+233662t^7-497816t^8+373320t^9-156658t^{10} \\
&\quad +36470t^{11}-3696t^{12}) , \\
P_7(t) &= (1-t)^{-11}(11440-22880t+20800t^2+51480t^3-231000t^4+456060t^5 \\
&\quad -581460t^6+518870t^7-329890t^8+147350t^9-44130t^{10} \\
&\quad +7980t^{11}-660t^{12}) , \\
P_8(t) &= (1-t)^{-11}(12870-50050t+109604t^2-98044t^3-142230t^4+629970t^5 \\
&\quad -1093180t^6+1187956t^7-877211t^8+443225t^9-147615t^{10} \\
&\quad +29325t^{11}-2640t^{12}) , \\
P_9(t) &= (1-t)^{-11}(11440-69718t+225698t^2-428120t^3+429000t^4+6270t^5 \\
&\quad -705034t^6+1142504t^7-1025320t^8+586190t^9-213290t^{10} \\
&\quad +45352t^{11}-4312t^{12}) , \\
P_{10}(t) &= (1-t)^{-11}(8008-66440t+269434t^2-656238t^3+1016400t^4-965184t^5 \\
&\quad +408870t^6+233662t^7-497816t^8+373320t^9-156658t^{10} \\
&\quad +36470t^{11}-3696t^{12}) , \\
P_{11}(t) &= (1-t)^{-11}(4368-43456t+199232t^2-544390t^3+970530t^4-1160049t^5 \\
&\quad +905707t^6-398761t^7+15995t^8+95650t^9-59318t^{10} \\
&\quad +16324t^{11}-1820t^{12}) , \\
P_{12}(t) &= 1820+560t , \\
P_{13}(t) &= 560+120t , \\
P_{14}(t) &= 120+16t , \\
P_{15}(t) &= 16+t , \\
P_{16}(t) &= 1 ,
\end{aligned} \tag{C.1}$$

The BRST operator acts “south-west” in the table. It is easily checked explicitly that these modules pair up in a way consistent with 1 being the only cohomology. This is of course a consequence of the fact that the purity constraint on r is the BRST variation of the one on $\bar{\lambda}$.

Consider for example the commutator $[\hat{\chi}^\alpha, \hat{\chi}^\beta]$. One possible term comes from the anti-commutator of the fermionic covariant derivatives. It contains $(\lambda\bar{\lambda})^{-4}\bar{\lambda}^2 r^2 \partial$ and transforms as (00100). Expanding $P_2(t)$ shows that the dimension of the modules occurring at $\bar{\lambda}^2 r^2$ is 12870. A more detailed calculation, performed by hand, or with the LiE code below, shows that these modules are $(00120) \oplus (01011)$. None of them contributes to (00100) when multiplied by a vector (10000), so this term vanishes for purely representation theoretical reasons. Such arguments in fact apply to any term in the equations (4.22) and (4.23), including those coming from N_{ab} acting on the $(\lambda\bar{\lambda})^{-p}$ prefactors.

```
##### some definitions #####
setdefault D5
lb=1X[0,0,0,1,0]
s=1X[0,0,0,1,0]
v=1X[1,0,0,0,0]
r(int n)=1X[0,0,0,n,0]
as(int n)=alt_tensor(n,s)
##### the positive part of a polynomial #####
pos_pol(pol p) =
{
  loc q=p;
  for i=1 to length(p) do
    if coef(p,i)<0 then q=q-p[i];
  fi;
od;
q
}
##### modules at r^m lambdabar^n #####
rr(int m,n)=
{
  if m==0 then
    r(n);
  else
    if n==0 then
      as(m);
```

```

else
    pos_pol(tensor(as(m),r(n))-tensor(v,tensor(as(m-1),r(n-1))));
fi;
fi;
}

```

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